A Study of Distributed Smart Sensor Networks [1] [2]

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Abstract

The ability to monitor or detect distributed physical stimuli as a means of alarming someone of danger or studying the behavior of nature has been of significant interest to the security community as well as the scientific community. The advances in measurement devices have reduced cost to the point where it is now viable to develop large-scale distributed sensing systems. Similar to the problems we face with the present Internet, these potential large-scale sensing systems involve vast amounts of localized information. However, the advances in processor technology allow for relatively low-cost, low-power, compact distributed processing integrated within these sensor devices, commonly referred to as *smart sensors.* Intelligent or smart sensors capable of parsing and filtering only the necessary or desired information, allow for efficient use of memory, precious wireless bandwidth, and battery power needed for the transfer of sensor information.

This paper describes a simple off-the-shelf implementation of smart sensors, capable of self-routing, and processing temperature information. Strong assumptions are made as to the topology of the network. Specifically, we develop a *distributed smart sensor network* where sensors, distributed throughout the network, are assumed to collect and process information independent of the network state and then forward information packets in a directed manner towards a *Home* base. Analysis is performed to obtain average routing times and average battery life expectancy of the modules.

Keywords: Smart Sensors, Intelligent Sensors, Distributed Sensing, Large Scale Networks, Ad-Hoc Networks

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[2] Preprint Technical Report revision A, Dartmouth College, Thayer School of Engineering, March 9, 2000.

1 Introduction

Consider a relatively remote site with no existing surveillance capabilities or security devices. Sources have recently indicated that this site is a potential *target* for malicious terrorist activity possibly with an objective to exploit vulnerabilities of the target or simply destroy the target (eg. a Local Area Network with low security but has recently become high profile). Inexpensive smart sensors capable of monitoring, processing, self-routing and wireless transmission are immediately dispersed throughout the entire area with the objective of self configuring an ad-hoc network for detecting unauthorized personnel from entering or corrupting the target site, as in Figure 1. Although this may sound like something straight out of a spy novel, there is and will always be a

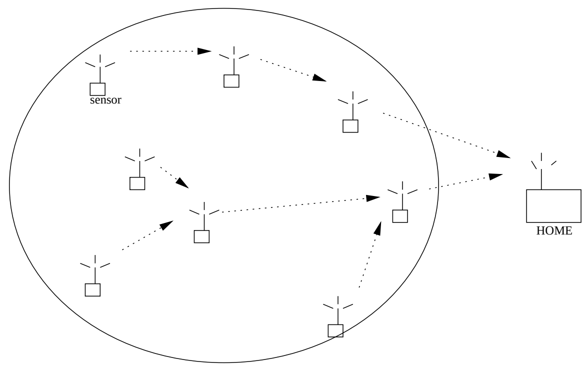


Figure 1: A distributed smart sensor network

need to gather information efficiently (low-cost) and safely (minimize the possibility of human harm or casualties).

This investigation will entail the design, prototype development, and testing of compact, inexpensive, wireless, smart sensors capable of dynamic self-routing, Internet connectivity, and localized signal processing. The concept of independent, localized signal processing of collected sensor data in a distributed network topology is defined as a *Distributed Smart Sensor Network* (DSSN). Of particular interest is the scalability issues associated with developing and integrating a large number of these distributed smart sensors creating a *very large scale network* (VLSN).

The topic of VLSN's has been addressed in [5, 4, 6] in terms of optimizing the capacity of these wireless networks. In this paper, we present an ad-hoc routing protocol, discuss the importance of using low-cost and modular hardware components in a design, and derive some performance measures for our routing algorithm and life expectancy of a module.

2 Architecture and Model

Due to the simplicity of common temperature sensors, we will initially consider a model involving the monitoring of temperature data. Typically, sensors of this sort will not be used for security purposes. However, one could visualize the scenario of utilizing this type of sensors to detect heat radiating off equipment or personnel from intruders entering a restricted area. In general, the modularity of our design (Figure 2) allows for flexibility in varying the type of sensor in our study and development. This implies that the exact type of sensor is not critical to the nature of the study.

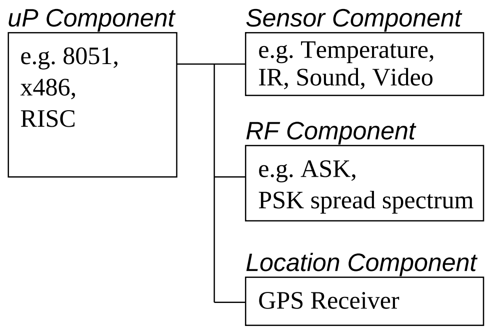


Figure 2: A sensor design exploiting modularity

2.1 System Level Infrastructure

To study Distributed Smart Sensor Networks, we propose the design of small, inexpensive, wireless units. Each unit is responsible for acquiring sensor data and then transmitting some form of the collected data through a RF wireless ad-hoc network. To prevent inherent dependencies, each unit in the network will be identical in design. After deployment into the network, each unit selects one of two states, *Module* or *Hub.* The current topology of the network will determine which state a unit is to select, while still allowing units to change state indefinitely on a periodic basis. The state of a unit determines its primary task and capabilities. The main objective of each unit, regardless of state, is to collect, process, and transmit data to a local *Home* or *Gateway.* A *Home* is defined as a location where a user is allowed to interact with the sensor network. (i.e. a laptop or PC terminal connecting to the DSSN.) A *Gateway* is simply an access node connecting the DSSN to an accessible LAN or possibly the Internet. Due to the ad-hoc nature of the DSSN, some form of transmission packet formatting and processing is required.

Through the use of modular *off-the-shelf* hardware and concise networking protocols and processing algorithms, we study and develop a dynamic network structure for distributed acquisition and processing of data measurements.

2.1.1 Hardware Module

All units are identical in regards to hardware components. Each unit as shown in Figure 3 is equipped with an off-the-shelf microprocessor, RF transceiver, GPS receiver, and temperature sensor. The concept of integrating intelligence into the sensor such as a processor, memory, and other peripheral circuitry is not new [7], and in general, considerable research has focused on flexible ASIC sensors [3]. However, as described above, each of our units is modular in design and consist of off-the-shelf components, allowing for reduced development cost and time (off-the-shelf parts are readily available), and ease of replacement (i.e. exchange type of sensor) or enhancement to meet specific mission project goals (i.e. replace RF with low power spread spectrum components to minimize the effect of RF jamming and detection).

The current configuration includes an Intel 80C51 8-bit microprocessor for controlling the unit. For communication between units and Home, we use an RF transceiver operating at a base frequency of 915Mhz, using an Amplitude Shift Keying (ASK) modulation scheme. A GPS receiver is used primarily for location determination and an addressing scheme, as described below in Section

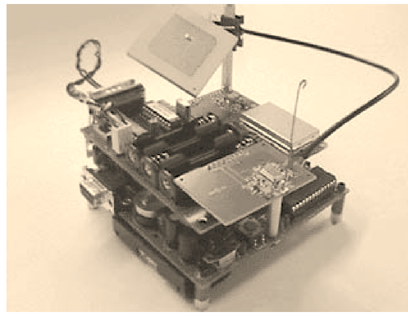


Figure 3: A typical module

2.1.2. Temperature sensing is performed with an 8-bit *iButton*$^{(\mathrm{TM})}$ sensor. In particular, this iButton technology[1], developed by Dallas Semiconductor, consists of easily replaceable small canisters (roughly the size of a button) with a variety of capabilities including stored memory, sensing, and network connectivity.

2.1.2 Protocol and Algorithm

To reduce network congestion and allow for efficient throughput, a proficient communication protocol must be developed. When developing this protocol, we rely on certain "strong" assumptions:

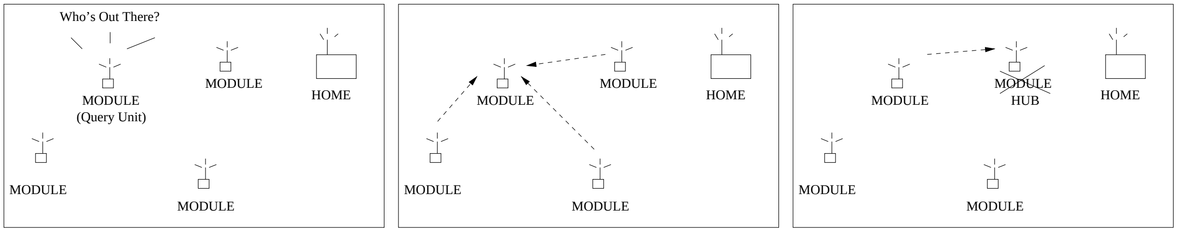
1. All units are allowed the ability to route data.
2. All sensor data is localized.(i.e. all data collection is performed at each unit, allowing each unit to require no knowledge of data collected by neighbor units.)
3. All *data communication* is one way (from unit to Home) while *control communication* is bidirectional.
4. Little or no communication is performed *Module-Module*. In general, all communication is either *Module-Hub, Hub-Hub,* or *Hub-Home.*

Utilizing the above assumptions, we present the *WOT-ACK* protocol:

On activation, a unit will utilize a GPS receiver to determine its relative position. Since the network may dynamically change state, we utilize the relative position of the unit as the unit's address location (synonymous with an IP address). For a given stationary unit, GPS position drift will be filtered out (i.e. positions within a certain radius define a unit's **fixed** address location). Utilizing the unit position information as an address location is essentially a form of geographical routing similar to the proposed Greedy Perimeter State Routing algorithm by Morris and Karp [9] and in the spirit of the work in [1].

Once position and address have been determined, a unit is ready to join the network and establish a communication route to a Home or Gateway. This process begins by sending a *Who's Out There?* (WOT) message, Figure 4(a). This message consist of the unit's address and an additional parameter, defined as the *Current Cost Function Value*(CCFV). The CCFV is a number representative of the "best" route metric.

[1]iButton Technology URL: http://www.ibutton.com



(a) Initial Query (b) Response (c) State Change

Figure 4: Initial Routing Algorithm

Neighboring units, either Modules or Hubs, who receive the WOT message take note of the CCFV and calculate their own *Cost Function Value* (CFV) to determine if they can provide a better route metric for the querying unit. If the neighboring unit's CFV is less than the CCFV, the neighboring unit responds within some finite random amount of time (random back-off) to the querying module with an acknowledgment (ACK) message as in Figure 4(b).

For a querying unit and a *potential route unit*, the optimal route towards Home requires that:

- The potential route unit be a Hub. (All information transfer of a Module must be via a Hub.)
- The potential route unit has the greatest number of available slots for data forwarding for a querying unit.
- The potential route unit is the closest of the competing route units to a Home.
- The potential route unit has an optimized absolute distance between a querying unit and itself, based on the range of the RF transceiver.

If the first criteria is not met, but the remaining three are, the querying unit can request that the potential route unit switch states from Module to Hub as depicted in Figure 4(c).

Calculating measures for the CFV will require some form of empirical testing and analysis. A possible weighted cost function CFV, currently being investigated, might take the following form:

$$CFV = C_1 * r + C_2 * x + C_3 * h + C_4 * s \tag{1}$$

where r is the absolute distance between query and neighbor units, x is the distance of neighbor unit to Home, h is the number of hops neighbor unit has to reach Home, s is the number of slots neighbor unit has available (if it is a Hub), and C_1, C_2, C_3 and C_4 are weighted constants calibrated for the topology of the network.

Using equation (1), we reduce bandwidth and transmission power consumption by only allowing a potential route unit to acknowledge a WOT message if it calculated a lower CFV than the CCFV. Initially, *all* units in receiving range will reply with their corresponding CFV since the CCFV is preliminarily set to a Worst Case Maximum. Included with this ACK reply message are the potential route unit's GPS location/address, and raw values for h and s.

Using these raw values, the querying unit checks for corrupted transmitted packets by recalculating CFVs for each received ACK message. If the recalculated CFV of a particular ACK message does not match the value contained in the corresponding ACK message, the querying unit disregards this ACK message assuming a packet has been corrupted.

Among all successfully received CFVs, the lowest CFV is selected and re-broadcasted in a new WOT message. The process of WOT and ACK messages repeats for all potential route units responding with lower CFV's until the lowest CFV is obtained for a querying unit. Once the route to a local Hub or Home is determined, a form of handshaking is performed between the querying unit and the newly selected Hub. All further information transfer from this querying unit will be performed through this Hub unless the network topology significantly changes requiring the querying unit to re-negotiate a *Hub-Module* connection.

2.1.3 Discussion on the Proposed Protocol

Allowing all units the ability to route packets results in *any* unit in the network becoming a viable link *(Hub)* to a Home. This idea has received scrutiny in the past, but unlike previous routing methods such as *distance vector* or *link state routing* [8], our scheme does not require storage of large routing tables. Since we assume the strong condition that all information transfer is unidirectional, each Module only needs to know its corresponding Hub, eliminating the need for large routing tables.

The WOT-ACK protocol with CFV described above allows for packet collisions to occur without creating false connections by insuring a valid connection between a querying unit and a potential route unit. Integrating the classic collision detection schemes with random back-off, typical in ethernet and Aloha [2], will reduce the effect of collisions and, along with CFVs, reduce the number of competing potential route units. Specific to the CFV is the nested "x-coordinate" direction (i.e. variable x in (1)) signifying distances towards Home, eliminating many potential route units that do not hold to the directed graph assumption.

3 Preliminary Analysis of Protocol

We now analyze our protocol generalizing the model in the previous section. In particular, we shall assume that "worst-case" acknowledgments shall occur from among the competing units attempting to acknowledge a querying unit.

Suppose there are initially N units that may potentially respond to an $N+1$ unit querying the network and consider a discrete time slot model (i.e. time is partitioned into integer slots $1, 2, \ldots$).

Let T_{GPSon} be the amount of time for the $N+1$ unit to acquire a valid GPS location measurement, T_{N+1} be the total amount of time it takes to establish a link for the $N+1$ unit given N units are available, and T_{minC} be the amount of time it takes for the $N+1$ unit to obtain the lowest cost function value from the N possible units. Assume zero time required for final handshaking with Hub and so we have

$$T_{N+1} = T_{GPSon} + T_{minC} . \quad (2)$$

Let T_{WOT} be the amount of time to send a "Who's Out There" message including back-off time, $T_{ACK,j,i}$ be the amount of time for the i^{th} competing unit in the j^{th} acknowledgment round, $\hat{N}$ be the number of rounds of WOT messages (equal to the number of rounds potential route units will respond).

Let p be the probability that a unit transmits an acknowledgment in a time slot, and so $1-p$ is the probability that a unit does not transmit an acknowledgment in a time slot. Thus, to successfully transmit an acknowledgment in a time slot, we need $p(1-p)^{n-1}$ where n is the number of potential route units.

Let $\tilde{N}_j$ be the number of competing units for the j^{th} round. We assume the "worst-case" algorithm where all units defined as competing units must acknowledge for each round j and that the $\tilde{N}_j$ are i.i.d..

From the above definitions and (2), we have

$$T_{N+1} \quad = \quad T_{GPSon} + \sum_{j=1}^{\hat{N}} T_{WOT} + \sum_{j=1}^{\hat{N}} \sum_{i=1}^{\tilde{N}_j} T_{ACK,j,i} \ . \tag{3}$$

Assume T_{GPSon} is a random variable taking on discrete values from $K_1, K_1+1, \ldots, K_2$ with equal probability, T_{WOT} is a random variable with equiprobable distribution corresponding to time slots $1, 2, 3, \ldots, K$, where K is the maximum number of backoff time slots before transmitting a WOT, and $\hat{N}$ and all the $\tilde{N}_j$ random variables taking on values $1, 2, \ldots, N$ with equal probability, where N is the maximum number of potential route units when introducing the $N+1$ unit, and all random variables are independent.

Recognize that since at least one unit acknowledgment is successfully received by the querying unit in each round, at most there are N rounds for $\hat{N}$ and so this distribution is reasonable.

Last, $T_{ACK,j,i}$ is a binomial random variable with distribution

$$f(k) = \{p(1-p)^{n-1}\}\{1-p(1-p)^{n-1}\}^{k-1} \quad \forall\ k = 1, 2, \ldots\ ,$$

representing transmission success, where n is the number of competing transmissions in the k^{th} time slot.

3.1 Time Intervals of a Module

Theorem 1 (Average Time to Route) *The average amount of time required for the $N+1$ unit to obtain a route through one of the potential N available units to be*

$$\begin{aligned} E[T_{N+1}] \quad = \quad & \frac{K_2+K_1}{2} + \frac{(N+1)(K+1)}{4} \\ & + \frac{N+1}{2}\left\{ \frac{(1-p)^{N+1} - (N+1)(1-p) + N}{Np^4(1-p)^{N-1}} - \frac{(1-p)(N+1)}{2p^2} \right\} \ . \end{aligned} \tag{4}$$

Proof of Theorem 1: See Appendix.

□

Now suppose that we add additional units to the network after the $N+1$ unit has entered the network. The $N+1$ unit may be a potential route with the $N+2$, $N+3$, $\ldots$, N_{max} unit. Let N_{pt} be the number of units that the $N+1$ unit is used as a potential route (i.e the $N+1$ will acknowledge to the N_{pt} units). Assume the distribution of N_{pt} is uniformly distributed over the discrete values $1, 2, \ldots, N_{max} - (N+2) + 1$. Similarly, we have $\tilde{N}_{max}$ and $\hat{N}_{max}$ as the random variables representing the number of units potentially responding to the additional units beyond the $N+1$ unit and the number of rounds the $N+1$ unit will acknowledge among the additional units, where both respective random variables have uniform distribution (i.e. $p(\hat{N}_{max} = i) = p(\tilde{N}_{max} = i) = \frac{1}{N_{max}} \quad \forall i = 1, 2, \ldots, N_{max}$).

Corollary 2 (Average Acknowledgement Time on $N+2, \ldots\ N_{max}$) *The average time spent by the $N+1$ unit potentially acknowledging the additional $N+2, \ldots, N_{max}$ units is*

$$E[T_{others}] = \frac{(1-p)^{N_{max}-N} - (N_{max}-N)(1-p) + N_{max} - N - 1}{(N_{max}-N-1)p^4(1-p)^{N_{max}-N-2}} - \frac{(1-p)(N_{max}-N)}{2p^2} .$$

Proof of Corollary 2: See Appendix.

□

Let T_{proc} be a random variable for the total amount of time to transmit (and acknowledge) sensor information forwarded. Equivalent to the percentage of time when processing in normal operating mode, Let N_{RF} be the number of compteing units for a given time slot, such that N_{max} is the maximum number of units and N_{RF} is a random variable taking on values $1, 2, \ldots, N_{max}$.

Corollary 3 (Average Data Processing Time) *If the above holds, then the average data processing time is*

$$E[T_{proc}] = 2 \cdot \left\{ \frac{(1-p)^{N_{max}+1} - (N_{max}+1)(1-p) + N_{max}}{N_{max}p^4(1-p)^{N_{max}-1}} - \frac{(1-p)(N_{max}+1)}{2p^2} \right\} .$$

Proof of Corollary 3: See Appendix.

□

3.2 Life Expectancy of a Module

Now consider calculating the average life expectancy of a unit. Specifically, we shall consider the $N+1$ unit entering a network such that at most N_{max} units will be in the network. We would like to calculate the average battery life of the $N+1$ unit considering all charge utilized in the initial routing, periodic sensing, and potential routing of additional units.

Let T be the total lifetime of a unit, T_{N+1} be the time required to obtain a route, $T_{otherroute}$ be the total time for any additional unit routing acknowledgments, and T_{norm} be the amount of time in normal operating mode (i.e. standard collection of temperature data).

Thus the average life expectancy of the $N+1$ unit is

$$E[T] = E[T_{N+1}] + E[T_{otherroute}] + E[T_{norm}] . \quad (5)$$

In order to calculate the third quantity on the right hand side of (5), we require removing the non-periodic charge of the initial start up of the $N+1$ unit. Specifically, since a battery is characterized in terms of charge (e.g. mA-hrs), we partition the events over time into non-periodic events and periodic events.

Consider a discrete time slot model. Let current dissipation of an element be $i_d[j]$ for time slot j, and so the total amount of charge dissipated between time slots 1 to k is

$$q[1,k] = \sum_{i=1}^{k} i_d[i] \ .$$

The non-periodic events shall involve the initial location of the GPS and the routing. We assume that these events occur only once in the life time of the unit.

Let Q_{batt} be the total amount of charge in the battery.

Let Q_{gpsup} be the total amount of charge needed for initial power up, Q_{route} be the total amount of charge for all RF routing transmissions and receptions (including initial routing of the $N+1$ unit, and additional routing acknowledgments for $N+2$ to N_{max} units), as well as $Q_{\mu Pup}$ be the amount of charge used by the microprocessor during the so-called power up sequence.

Let Q_{norm} be the total amount of charge allowed for periodic events, and so we have

$$Q_{norm} = Q_{batt} - Q_{gpsup} - Q_{route} - Q_{\mu Pup} \ ,$$

and consequently, we have

$$\begin{aligned} E[T_{norm}] &\simeq \frac{E[Q_{batt} - Q_{gpsup} - Q_{route} - Q_{\mu Pup}]}{E[i_{norm}]} \\ &= \frac{Q_{batt} - E[Q_{gpsup}] - E[Q_{route}] - E[Q_{\mu Pup}]}{E[i_{norm}]} \ , \end{aligned} \tag{6}$$

where i_{norm} is the amount of current utilized in the periodic "normal" operating event of the unit. From (5) and (6), we have

$$E[T] = E[T_{N+1}] + E[T_{otherroute}] + \frac{Q_{batt} - E[Q_{gpsup}] - E[Q_{route}] - E[Q_{\mu Pup}]}{E[i_{norm}]} \ . \tag{7}$$

Let T_{sense} be a fixed constant number of slots of a single sensing period and T_{proc} is the number of time slots in a transmit and acknowledgment scheme such that no collisions occurred.
Let

$$\begin{aligned} T_{avgroute} &= \frac{K_2 + K_1}{2} + \frac{(N+1)(K+1)}{4} \\ &\quad + \frac{N+1}{2} \left\{ \frac{(1-p)^{N+1} - (N+1)(1-p) + N}{Np^4(1-p)^{N-1}} - \frac{(1-p)(N+1)}{2p^2} \right\} \ . \end{aligned}$$

Let

$$T_{avgothers} = \frac{(1-p)^{N_{max}-N} - (N_{max}-N)(1-p) + N_{max} - N - 1}{(N_{max} - N - 1)p^4(1-p)^{N_{max}-N-2}} - \frac{(1-p)(N_{max}-N)}{2p^2} \ .$$

We shall define a constant T_{normal}, in terms of battery charge, number of network units, and the various component current consumptions. Specifically, let

$$T_{normal} = \frac{Q_{batt} - \hat{Q}_1 - \hat{Q}_2 - \hat{Q}_3}{\hat{i}} \ ,$$

where

$$\hat{Q}_1 = \frac{(K_2 + K_1) i_{GPSon}}{2} ,$$

$$\hat{Q}_2 = \frac{(N+1) i_{RFtx}}{2} + \frac{(N+1)^2 i_{RFrx}}{4} + \frac{N_{max} - N}{2} \frac{(N_{max}+1)^2 (i_{RFtx} + i_{RFrx})}{4} ,$$

$$\hat{Q}_3 = (T_{avgroute} + T_{avgothers}) i_{\mu Pon} ,$$

$$\hat{i} = i_{off} + \frac{i_{on} - i_{off}}{T_{sense}} E[T_{proc}] ,$$

$$i_{on} = i_{\mu Pon} + \frac{i_{RFrx} + i_{RFtx}}{2} + i_{sensON} ,$$

$$i_{off} = i_{\mu Poff} + i_{RFoff} + i_{GPSoff} + i_{sensOFF} ,$$

i_{RFtx} is the current used by the RF component when transmitting, i_{RFrx} is the current used by the RF component when receiving, $i_{\mu Pon}$ is the current used by the microprocessor when computing and operating, $i_{\mu Poff}$ is the current used by the microprocessor when the unit is in sleep mode, i_{GPSon} is the current used by the GPS unit on initial acquisition, i_{GPSoff} is the current used by the GPS when not acquiring location, i_{sensON} is the current used by the sensor when acquiring data, and $i_{sensOFF}$ is the current used by the sensor when not acquiring data.

Theorem 4 (Average Life Expectancy of a Unit) *If $T_{sense} \geq T_{proc}$, then the life expectancy of the $N+1$ unit given that a total N_{max} units will enter the network is $T_{avgroute} + T_{avgothers} + T_{normal}$ time slots.*

Before proving the above theorem, we state the following useful corollary.

Corollary 5 *If $T_{sense} \geq T_{proc}$, then $E[T_{norm}] = T_{normal}$.*

Proof of Corollary 5: See Appendix.

□

Proof of Theorem 4:

Utilizing (7) and invoking Theorem 1, Corollary 2, and Corollary 5, we get the first, second, and third quantities on the right hand side in Theorem 4, respectively.

□

4 Status and Future Work

The project involves a level of testing off-the-shelf equipment. Currently, four prototype modules have been placed together with some level of self-routing connectivity tested. Integration of the GPS location addressing and iButton data collection are in the works.

Clearly, we must continue developing, testing, and implementing our routing algorithm. Refinement and confirmation of accuracy of our preliminary analysis will progress as our algorithms are developed. Based on our "strong" assumptions and inexpensive off-the-shelf equipment we shall continue to study the effects of large scale distributed networks.

Appendix

Proof of Theorem 1:

From (3), we have

$$E[T_{N+1}] = E[T_{GPSon}] + E[\sum_{j=1}^{\hat{N}} T_{WOT}] + E[\sum_{j=1}^{\hat{N}} \sum_{i=1}^{\tilde{N}_j} T_{ACK,j,i}] \ . \tag{8}$$

Since T_{GPSon} is a random variable taking on discrete values from $K_1, K_1+1, \ldots, K_2$ with equal probability, for the first quantity on the right hand side of (8), we have the trivial result

$$E[T_{GPSon}] = \frac{K_2 + K_1}{2} \ . \tag{9}$$

For the second part on the right hand side of (8), we have

$$E[\sum_{j=1}^{\hat{N}} T_{WOT}] = E[E[\sum_{j=1}^{\hat{N}} T_{WOT} | \hat{N}]] \ , \tag{10}$$

and since

$$\begin{aligned} E[\sum_{j=1}^{\hat{N}} T_{WOT} | \hat{N} = n] &= E[\sum_{j=1}^{n} T_{WOT}] \\ &= nE[T_{WOT}] \\ &= n\frac{K+1}{2} \ , \end{aligned}$$

using (10), we have

$$\begin{aligned} E[\sum_{j=1}^{\hat{N}} T_{WOT}] &= E[\hat{N}\frac{K+1}{2}] \\ &= \frac{(N+1)(K+1)}{4} \ . \end{aligned} \tag{11}$$

For the third quantity on the right hand side of (8), we have

$$E[\sum_{j=1}^{\hat{N}} \sum_{i=1}^{\tilde{N}_j} T_{ACK,j,i}] = E[E[\sum_{j=1}^{\hat{N}} \sum_{i=1}^{\tilde{N}_j} T_{ACK,j,i} | \tilde{N} \text{ and } \hat{N}]] \ . \tag{12}$$

Examining the inner expectation of (12) , we have

$$E[\sum_{j=1}^{\hat{N}} \sum_{i=1}^{\tilde{N}_j} T_{ACK,j,i} | \tilde{N} = n_2 \text{ and } \hat{N} = n_1] = E[\sum_{j=1}^{n_1} \sum_{i=1}^{n_2} T_{ACK,j,i}]$$

$$
\begin{aligned}
&= \sum_{j=1}^{n_1} E[\sum_{i=1}^{n_2} T_{ACK,j,i}] \\
&= n_1 E[\sum_{i=1}^{n_2} T_{ACK,j,i}] \\
&= n_1 \sum_{k=1}^{\infty} \sum_{i=0}^{n_2-1} k\{p(1-p)^i\}\{1-p(1-p)^i\}^{k-1} \\
&= n_1 \sum_{i=0}^{n_2-1} \frac{p(1-p)^i}{(p(1-p)^i)^2} \\
&= n_1 \sum_{i=0}^{n_2-1} \frac{1}{p(1-p)^i} \\
&= n_1 \frac{1}{p} \cdot \frac{1-(\frac{1}{1-p})^{n_2}}{1-\frac{1}{1-p}} \\
&= n_1 \frac{1}{p} \cdot \frac{(1-p)-(\frac{1}{1-p})^{n_2-1}}{-p} \\
&= n_1 \frac{1}{p^2} \cdot \frac{1-(1-p)^{n_2}}{(1-p)^{n_2-1}} \quad .
\end{aligned}
$$

Using the above result and (12), we have

$$
\begin{aligned}
E[\sum_{j=1}^{\hat{N}} \sum_{i=1}^{\tilde{N}_j} T_{ACK,j,i}] &= E[\hat{N} \cdot \frac{1-(1-p)^{\tilde{N}_j}}{p^2(1-p)^{\tilde{N}_j-1}}] \\
&= E[\hat{N}] \cdot E[\frac{1-(1-p)^{\tilde{N}_j}}{p^2(1-p)^{\tilde{N}_j-1}}] \\
&= \frac{N+1}{2} \cdot E[\frac{1-(1-p)^{\tilde{N}}}{p^2(1-p)^{\tilde{N}-1}}] \\
&= \frac{N+1}{2} \sum_{i=1}^{N} \frac{1}{N} \cdot i \cdot \frac{1-(1-p)^i}{p^2(1-p)^{i-1}} \\
&= \frac{N+1}{2} \cdot \frac{1}{Np^2} \left\{ \sum_{i=1}^{N} i \frac{1}{(1-p)^{i-1}} - \sum_{i=1}^{N} i(1-p) \right\} \\
&= \frac{N+1}{2} \cdot \frac{1}{Np^2} \left\{ \frac{1-(N+1)(\frac{1}{1-p})^N + N(\frac{1}{1-p})^{N+1}}{[1-(\frac{1}{1-p})]^2} - \frac{(1-p)(N+1)N}{2} \right\} \\
&= \frac{N+1}{2} \left\{ \frac{(1-p)^{N+1}-(N+1)(1-p)+N}{Np^4(1-p)^{N-1}} - \frac{(1-p)(N+1)}{2p^2} \right\} \quad . \qquad (13)
\end{aligned}
$$

By combining (9), (11), and (13), we get (4) and we are done.

□

Sketch Proof of Corollary 2:

Recall that N_{pt} takes on values $1, 2, \ldots, N_{max} - N - 1$ with equal probability. Recognize that

$$E[T_{others}] = E[E[T_{others}|N_{pt}]] ,$$

and since

$$\begin{aligned} E[T_{others}|N_{pt} = n] &= \sum_{j=1}^{n} \sum_{k=1}^{\infty} k\{p(1-p)^{j-1}\}\{1 - p(1-p)^{j-1}\}^{k-1} \\ &= \sum_{j=1}^{n} \frac{1}{p(1-p)^{j-1}} \\ &= \frac{1-(1-p)^n}{p^2(1-p)^{n-1}} , \end{aligned}$$

and utilizing the techniques found in the proof of Theorem 1, we have

$$\begin{aligned} E[T_{others}] &= E[\frac{1-(1-p)^{N_{pt}}}{p^2(1-p)^{N_{pt}-1}}] \\ &= \frac{1}{(N_{max} - N - 1)} \sum_{i=1}^{N_{max}-N-1} i \cdot \frac{1-(1-p)^i}{p^2(1-p)^{i-1}} \\ &= \frac{(1-p)^{N_{max}-N} - (N_{max} - N)(1-p) + N_{max} - N - 1}{(N_{max} - N - 1)p^4(1-p)^{N_{max}-N-2}} - \frac{(1-p)(N_{max} - N)}{2p^2} . \end{aligned}$$

□

Sketch Proof of Corollary 3:

Since there is an acknowledgment for every transmission, we simple multiple the average time to transmit by a factor of 2. Thus, we have

$$E[T_{proc}] = E[E[T_{proc}|N_{RF}]]$$

Since

$$\begin{aligned} E[T_{proc}|N_{RF} = n] &= 2 \cdot \sum_{j=1}^{n} \sum_{k=1}^{\infty} k\{p(1-p)^{j-1}\}\{1 - p(1-p)^{j-1}\}^{k-1} \\ &= 2 \cdot \sum_{j=1}^{n} \frac{1}{p(1-p)^{j-1}} \\ &= 2 \cdot \frac{1-(1-p)^n}{p^2(1-p)^{n-1}} , \end{aligned}$$

and utilizing the techniques found in the proof of Theorem 1, we have

$$E[T_{proc}] = 2E[\frac{1-(1-p)^{N_{RF}}}{p^2(1-p)^{N_{RF}-1}}]$$

$$= \quad 2 \cdot \left\{ \frac{(1-p)^{N_{max}+1} - (N_{max}+1)(1-p) + N_{max}}{N_{max} p^4 (1-p)^{N_{max}-1}} - \frac{(1-p)(N_{max}+1)}{2p^2} \right\} .$$

□

Proof of Corollary 5:
Recognize that the second part of (6) is

$$\begin{aligned} E[Q_{gpsup}] &= E[T_{GPSon} i_{GPSon}] \\ &= E[T_{GPSon}] i_{GPSon} \\ &= \frac{(K_2 + K_1) i_{GPSon}}{2} . \end{aligned} \quad (14)$$

Now we examine the third part of (6). Since a route involves a WOT RF transmissions for the $N+1$ unit and the number of ACK transmissions for any additional unit and since the $N+1$ unit may be a potential route with the $N+2$, $N+3$, ..., N_{max} unit with probability distribution $\frac{1}{N_{max}-N-1}$, we have

$$\begin{aligned} E[Q_{route}] &= E[\hat{N} i_{RFtx}] + E[\tilde{N}\hat{N} i_{RFrx}] + E[N_{pt}\tilde{N}_{max}\hat{N}_{max}(i_{RFtx} + i_{RFrx})] \\ &= \frac{(N+1) i_{RFtx}}{2} + +E[\tilde{N}\hat{N} i_{RFrx}] + E[N_{pt}\tilde{N}_{max}\hat{N}_{max}(i_{RFtx} + i_{RFrx})] \\ &= \frac{(N+1) i_{RFtx}}{2} + \frac{(N+1)^2 i_{RFrx}}{4} + E[N_{pt}\tilde{N}_{max}\hat{N}_{max}(i_{RFtx} + i_{RFrx})] \\ &= \frac{(N+1) i_{RFtx}}{2} + \frac{(N+1)^2 i_{RFrx}}{4} + \frac{N_{max} - N}{2} \frac{(N_{max}+1)^2 (i_{RFtx} + i_{RFrx})}{4} . \end{aligned} \quad (15)$$

Now we examine the fourth part of (6). Since the processor is operating throughout all the initial route time and the other additional units added to the network after $N+1$ assuming the $N+1$ unit will respond to some number of these additional units, using Theorem 1 and Corollary 2, we have

$$\begin{aligned} E[Q_{\mu Pon}] &= E[(T_{N+1} + T_{otherroute}) i_{\mu Pon}] \\ &= (T_{avgroute} + T_{avgothers}) i_{\mu Pon} . \end{aligned} \quad (16)$$

We now evaluate $E[i_{norm}]$ in (6). Recognize that since $T_{proc} < T_{sense}$, then

$$E[T_{proc}] < T_{sense} . \quad (17)$$

Recall the definitions for i_{on} and i_{off}, such that

$$i_{on} = i_{\mu Pon} + \frac{i_{RFrx} + i_{RFtx}}{2} + i_{sensON} ,$$

$$i_{off} = i_{\mu Poff} + i_{RFoff} + i_{GPSoff} + i_{sensOFF} .$$

Since normal operations occur only once per period T_{sense}, we separate current and operating intervals, use (17) and the definition for i_{on} and i_{off}. We have

$$E[i_{norm}] \quad = \quad E[T_{proc} i_{on} + T_{not-operating} i_{off}]$$

$$
\begin{aligned}
&= \frac{E[T_{proc}]i_{on} + E[T_{not-operating}]i_{off}}{T_{sense}} \\
&= \frac{E[T_{proc}]i_{on} + E[T_{sense} - T_{proc}]i_{off}}{T_{sense}} \\
&= i_{off} + \frac{i_{on} - i_{off}}{T_{sense}} E[T_{proc}] \ ,
\end{aligned}
$$

and we are done.

□

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